

1 Additional Problems

1.1 Sylow Theorems

1. Do one of the following.
 - (a) Prove no group of order p^2q , where p and q are prime, is simple.
 - (b) Let G be a group of order $p^l m$. Our textbook (Gallian) contains an argument that G contains a subgroup of order p^r for every integer $1 \leq r \leq l$. Finish this argument by proving exercise 45 in chapter 10 of Gallian. That is, Let N be a normal subgroup of a group G . Use property 7 of Theorem 10.2 to prove every subgroup of G/N has the form H/N where H is a subgroup of G .

2. Do one of the following.
 - (a) Prove the only simple groups of order less than 60 are groups of prime order.
 - (b) Classify all groups of order 18.
 - (c) Prove no group of order 224 is simple.